Blob detection

370: Intro to Computer Vision

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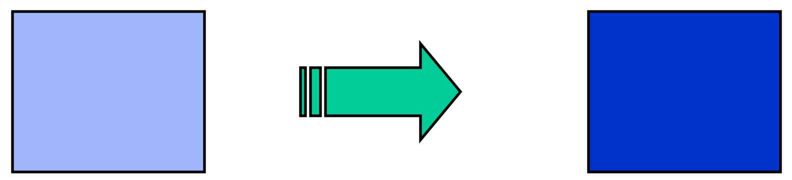
Invariance and covariance (a quick review…)

Invariance: transformations *do not change* the corner locations

Covariance or Equivariance: transformations change corner locations *in a predictable way*

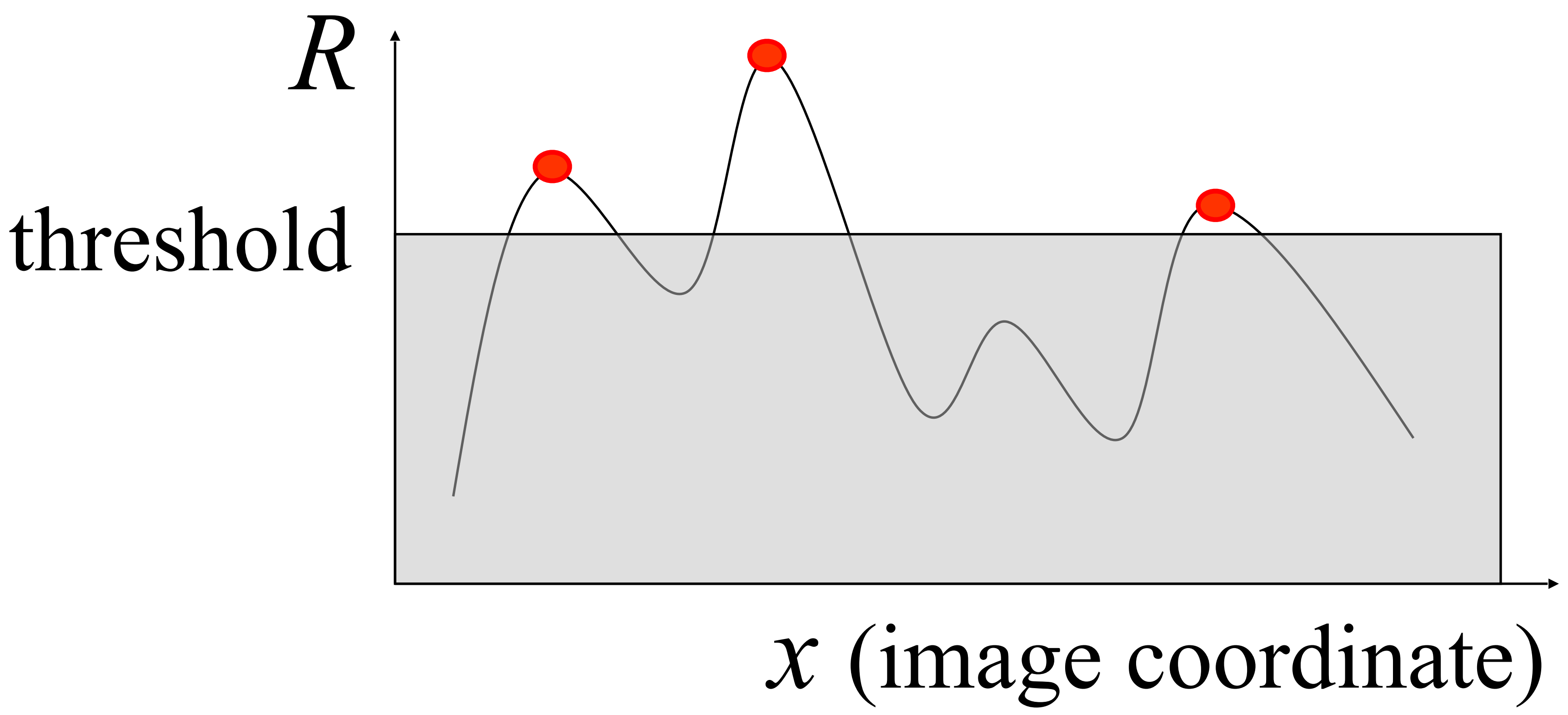
We want corners to be *invariant* to photometric transformations and *covariant* to geometric transformations

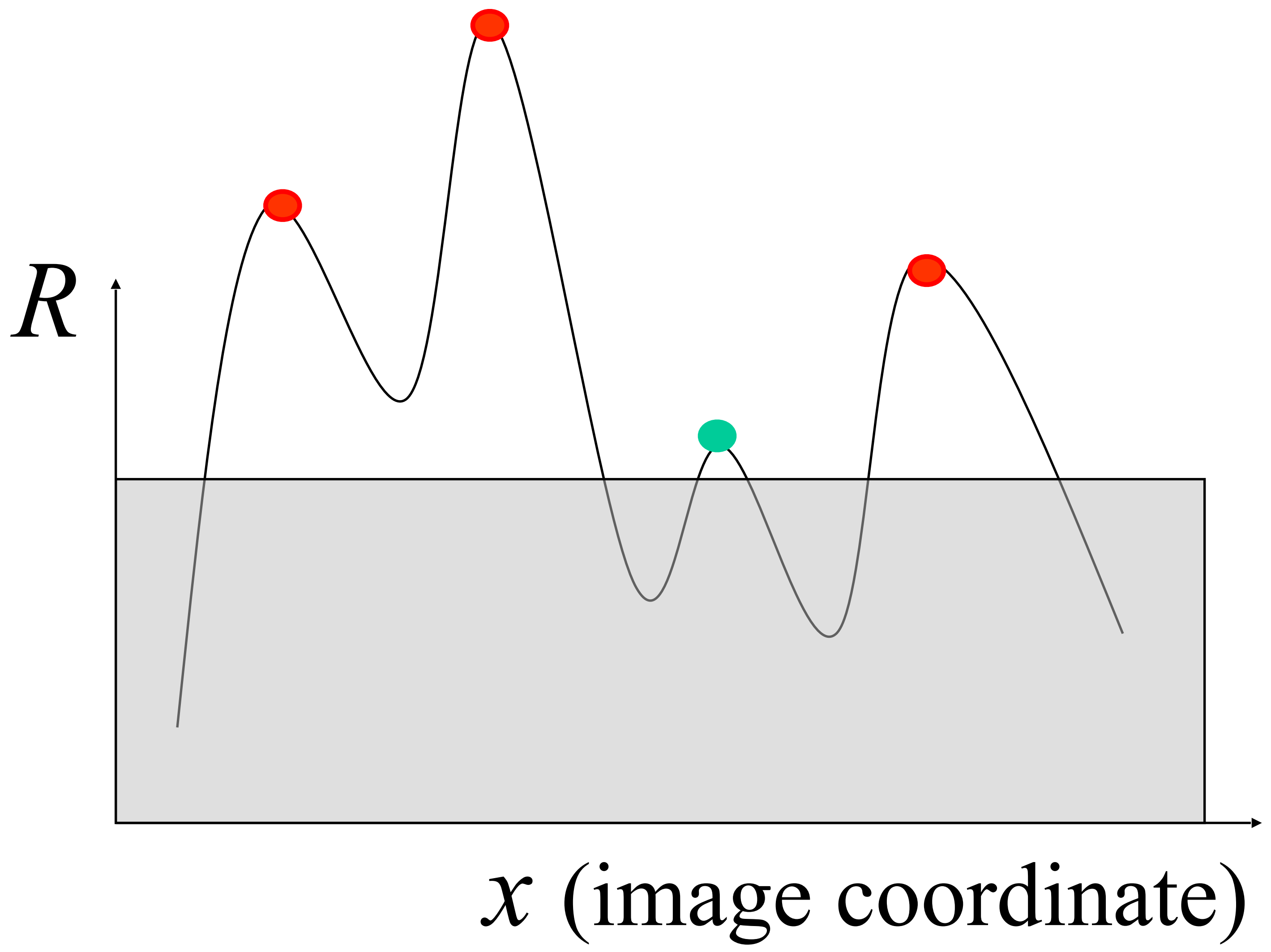
Affine intensity change



$$I \rightarrow a\,I + b$$

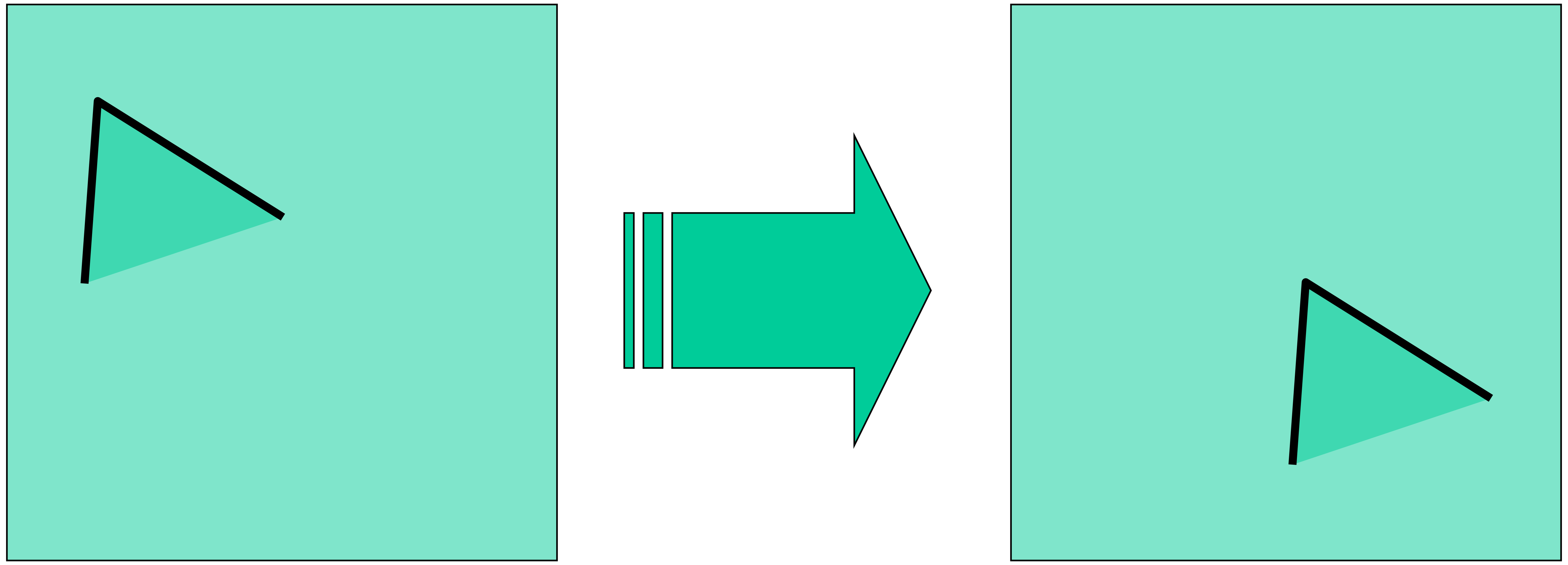
- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a\,I$





Corner location is partially invariant to affine intensity change

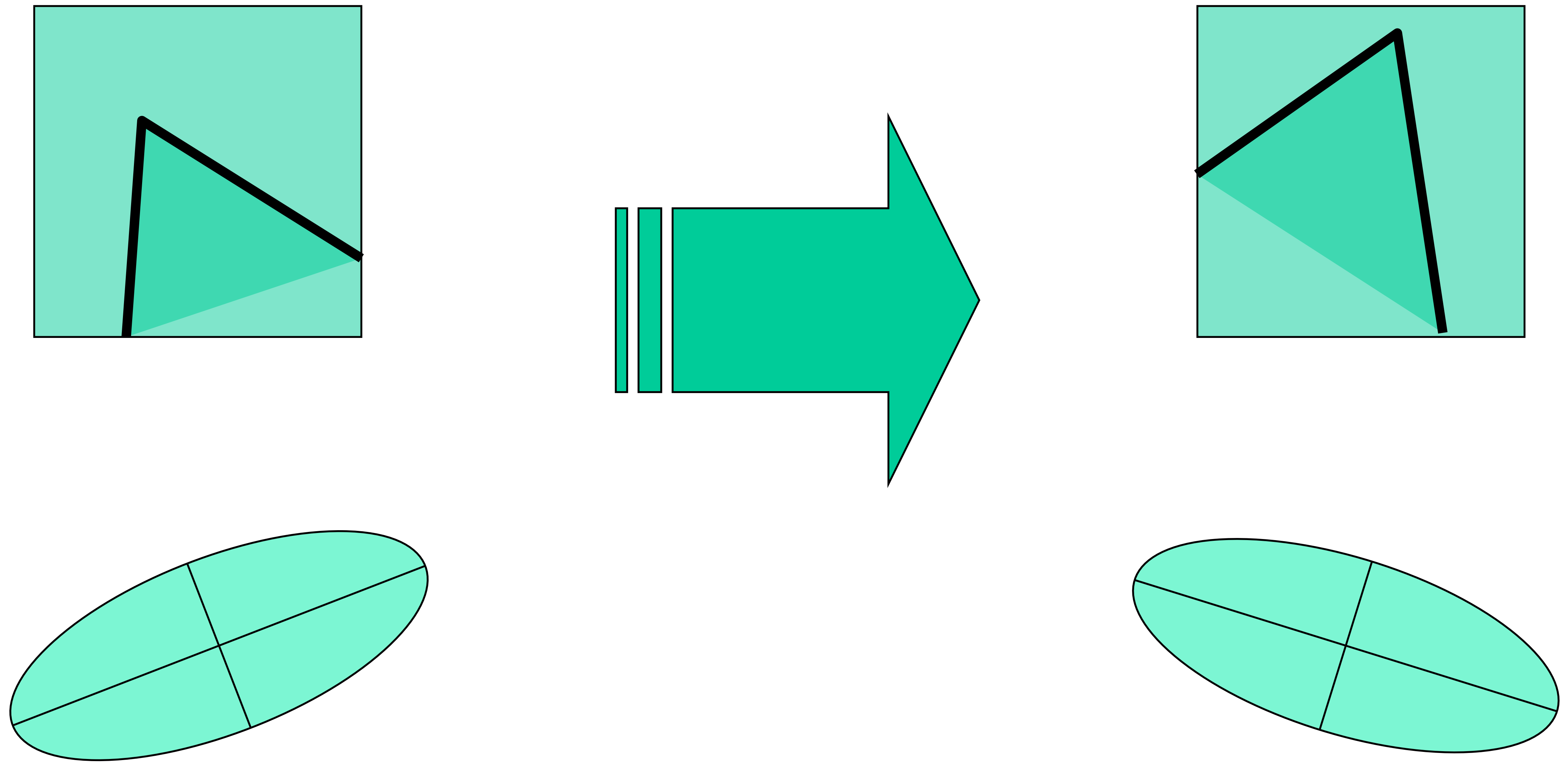
Translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

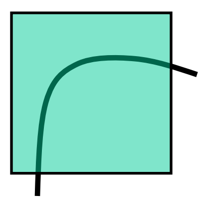
Rotation



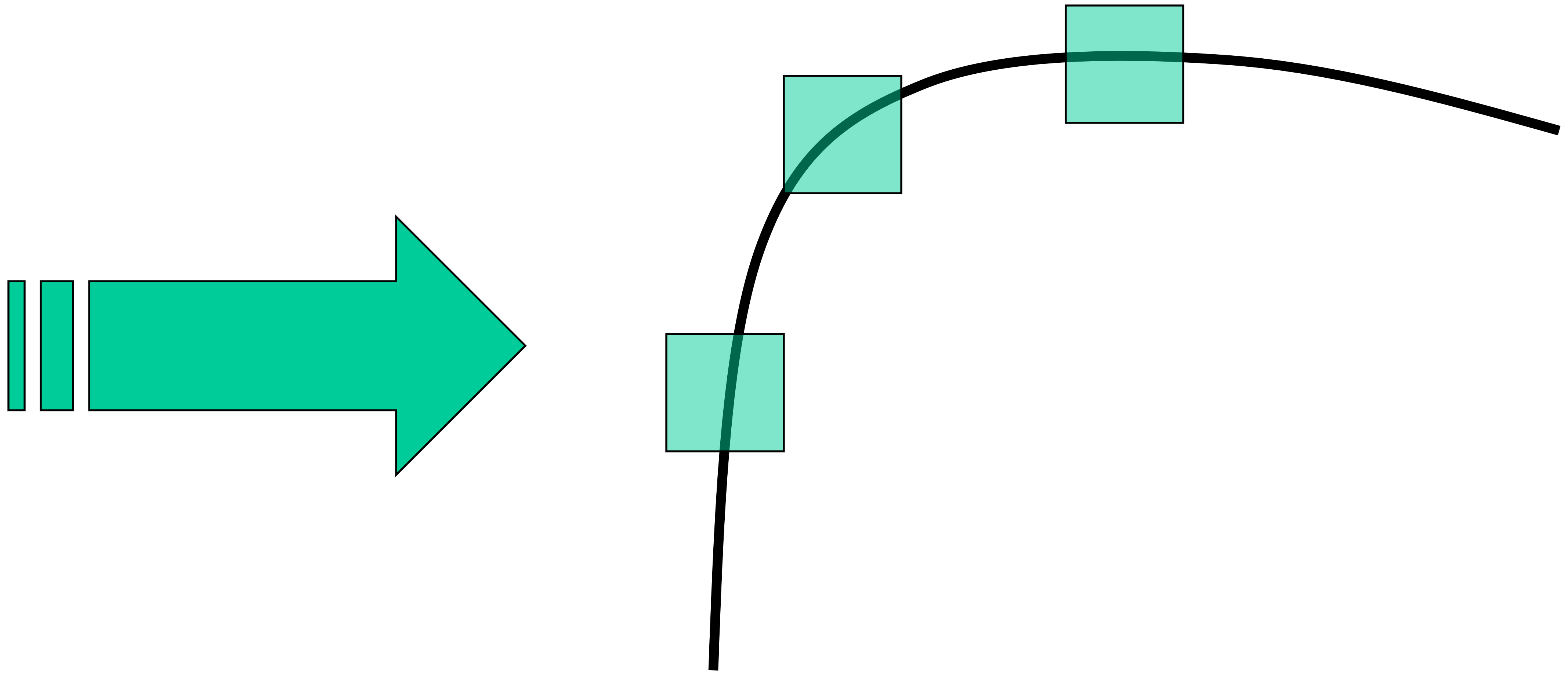
Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Scaling



Corner



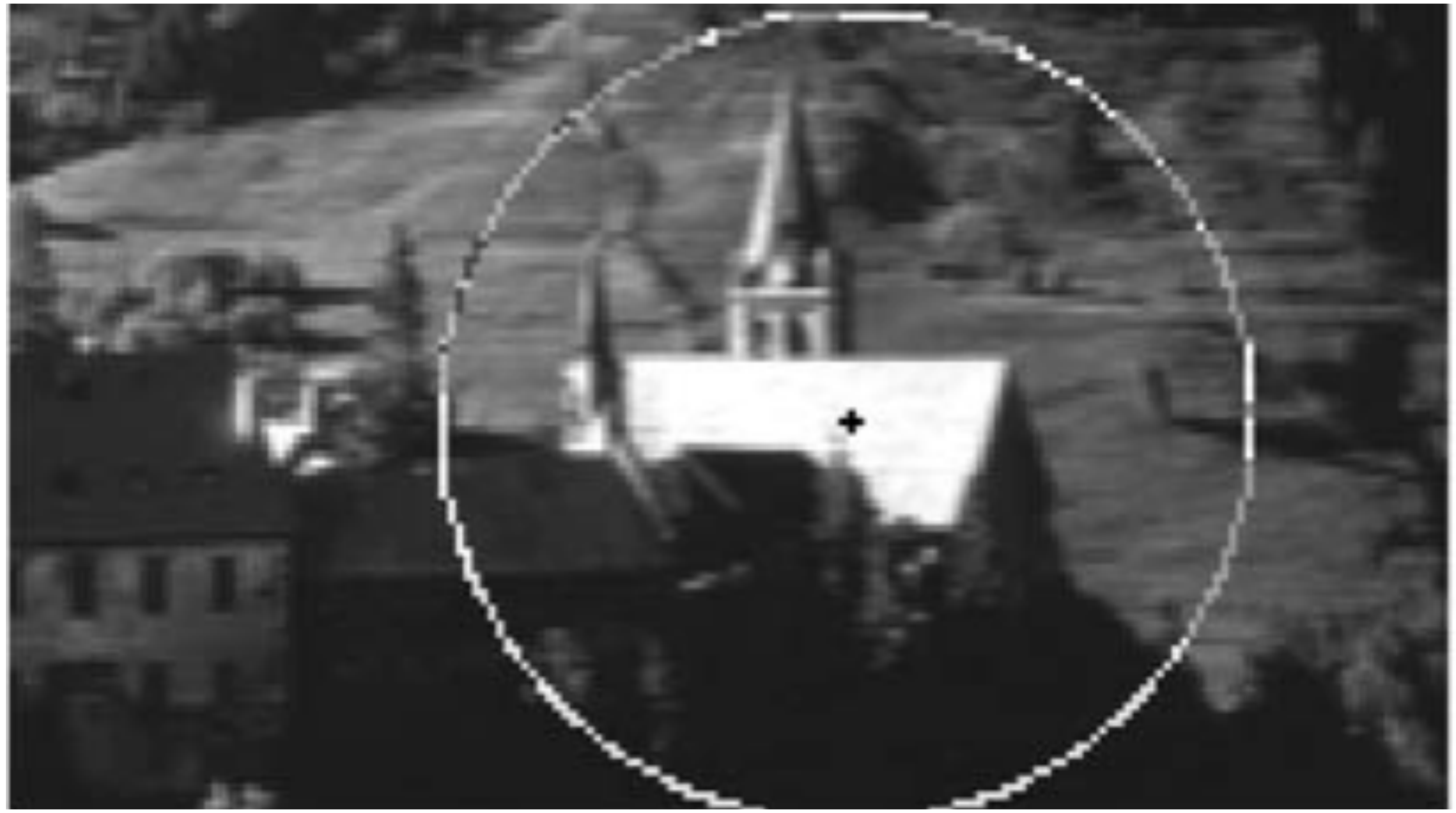
All points will be classified as edges

Corner detection is sensitive to the image scale!

Source: L. Lazebnik

Feature detection with scale selection

We want to extract features with characteristic scale that matches the image transformation such as scaling and translation

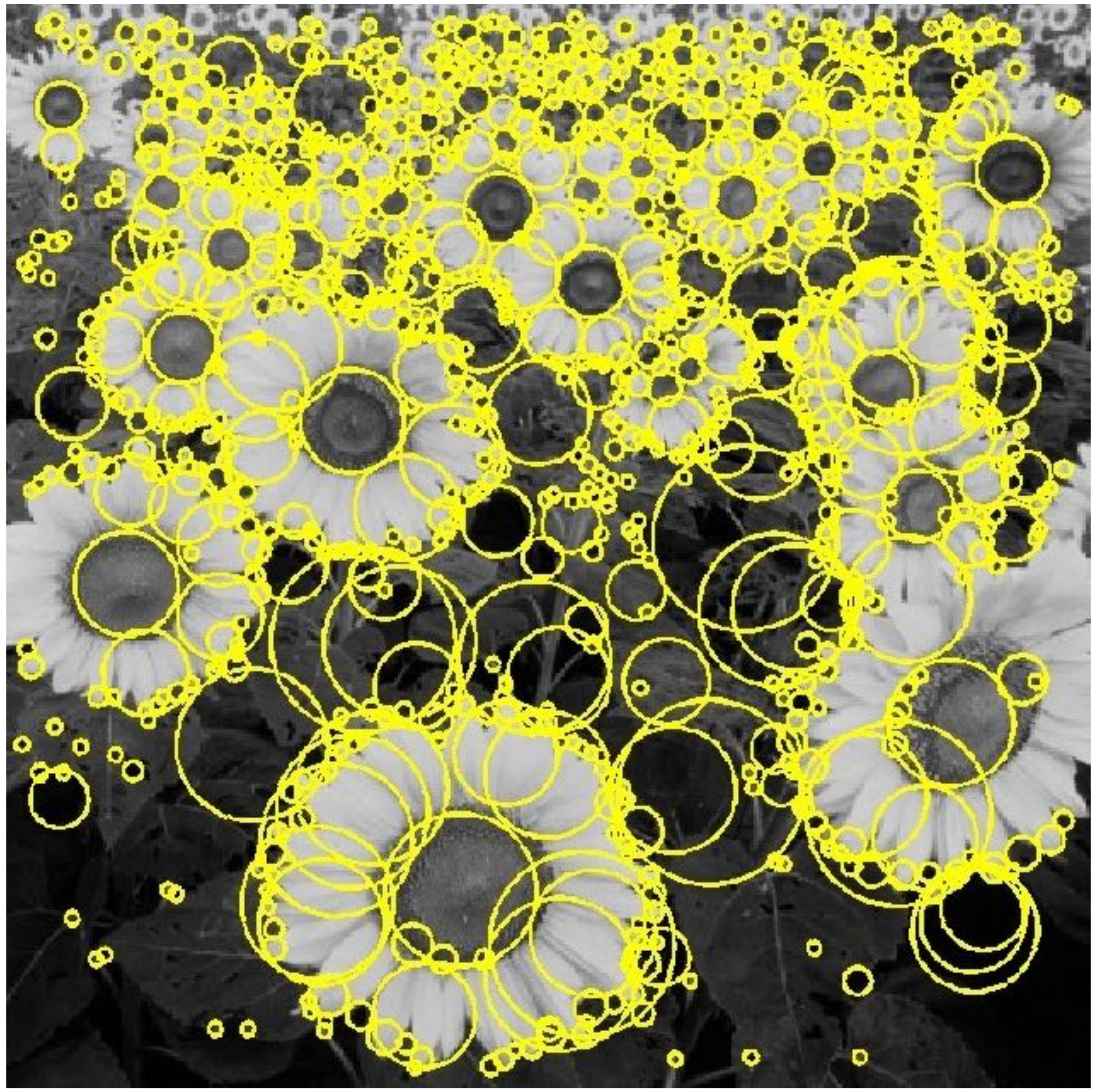


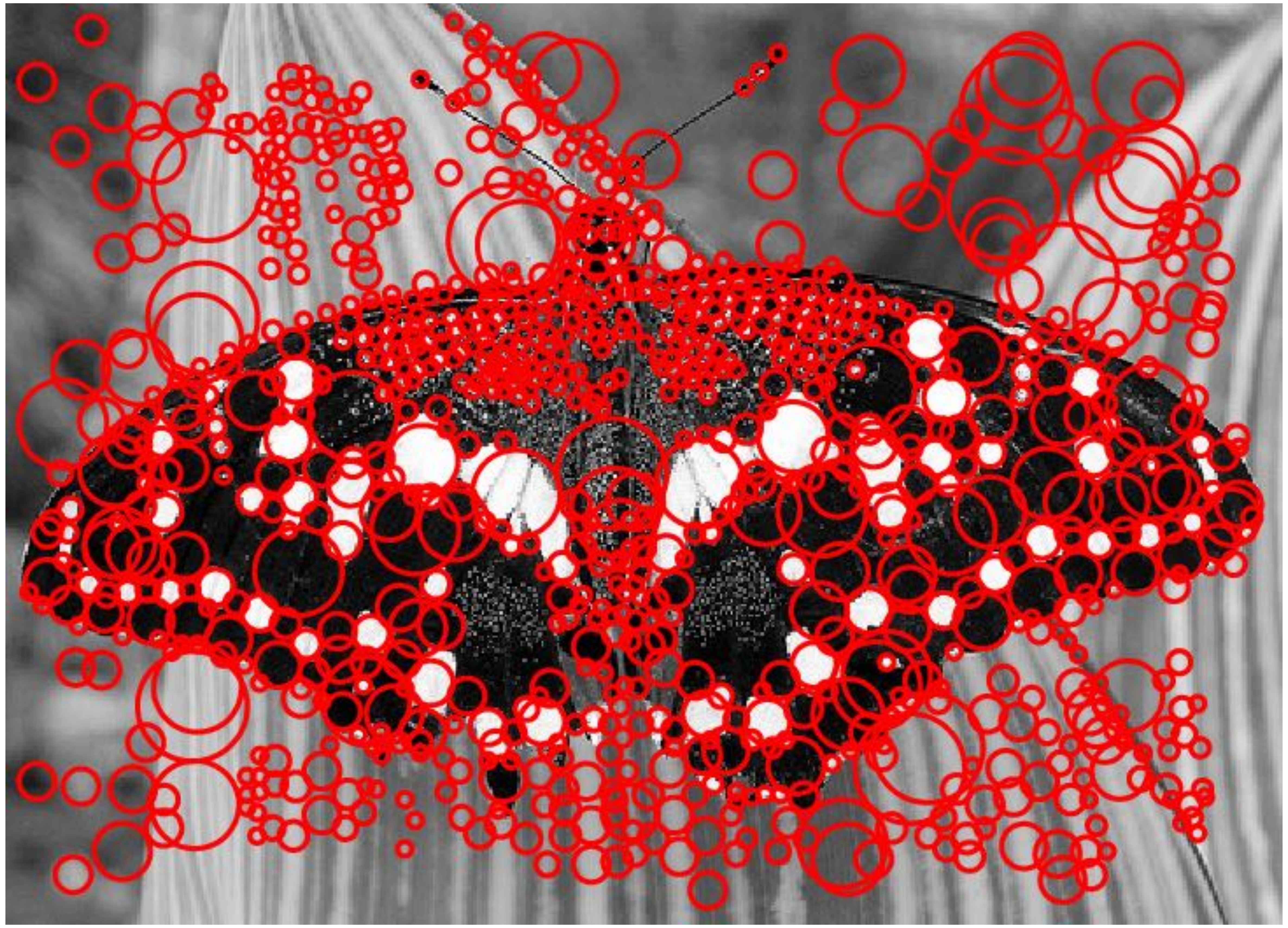
Matching regions across scales

Source: L. Lazebnik

Scale covariant features

Scale “covariance”

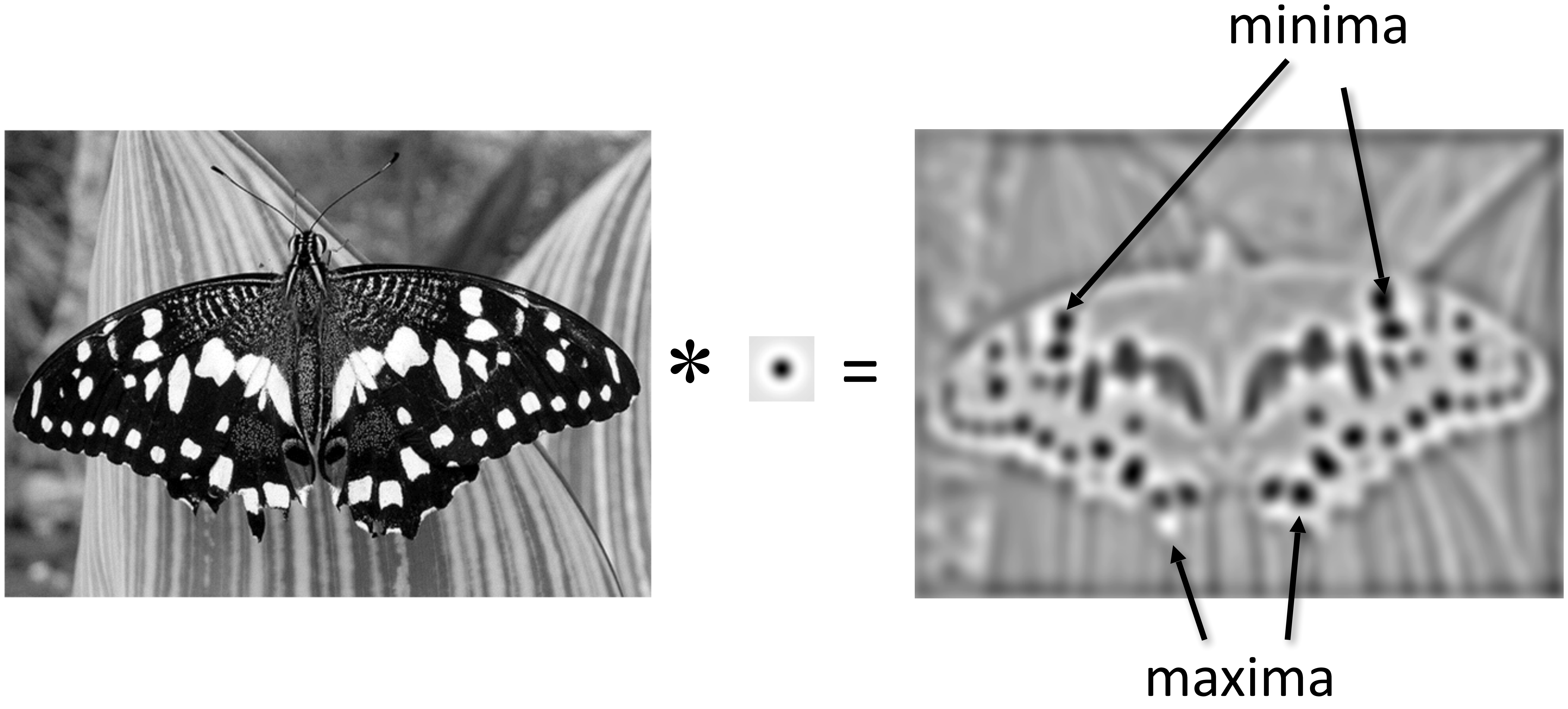




Source: L. Lazebnik

Blob detection: basic idea

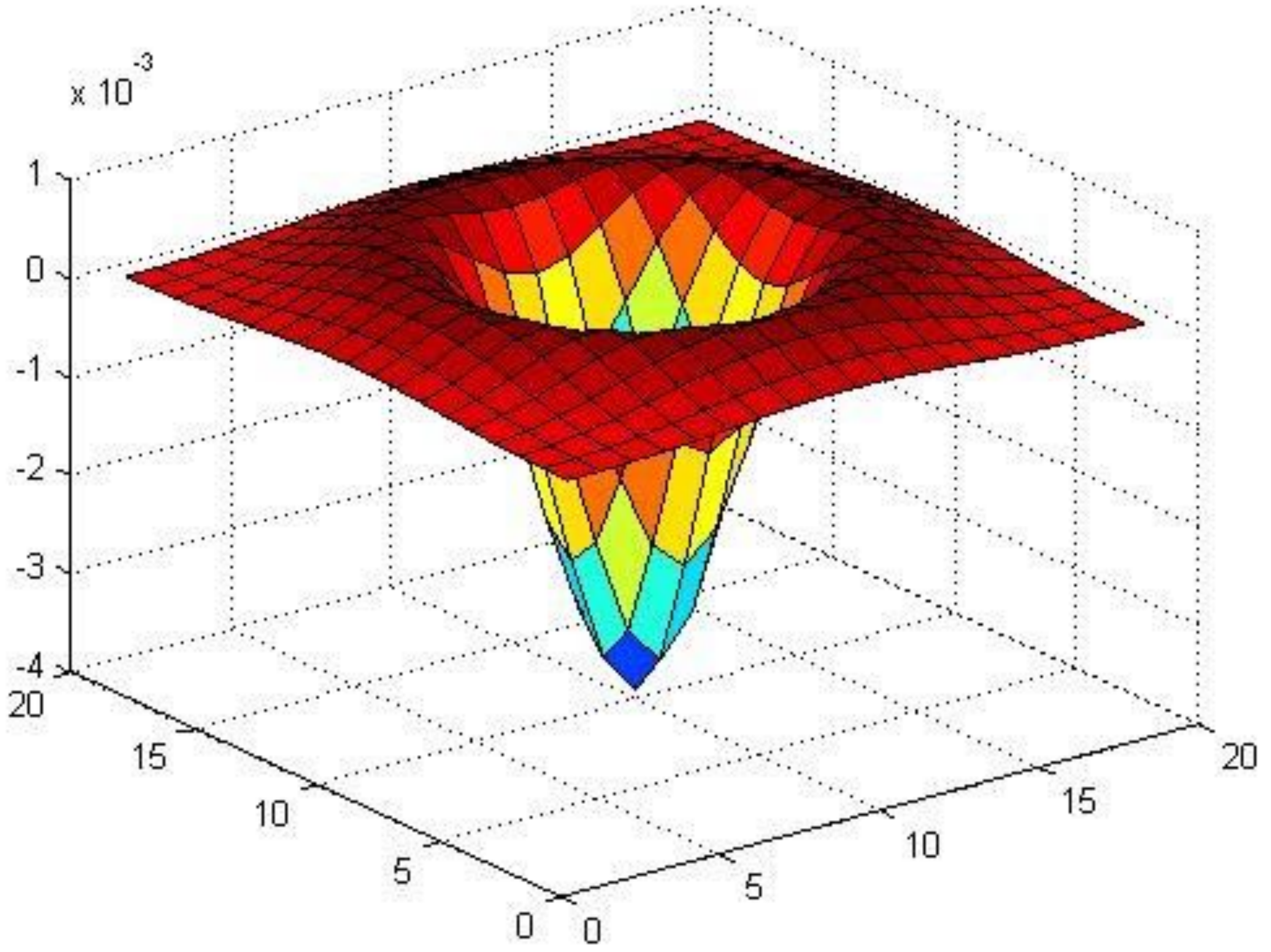
Find maxima *and minima* of blob filter response in space *and scale*

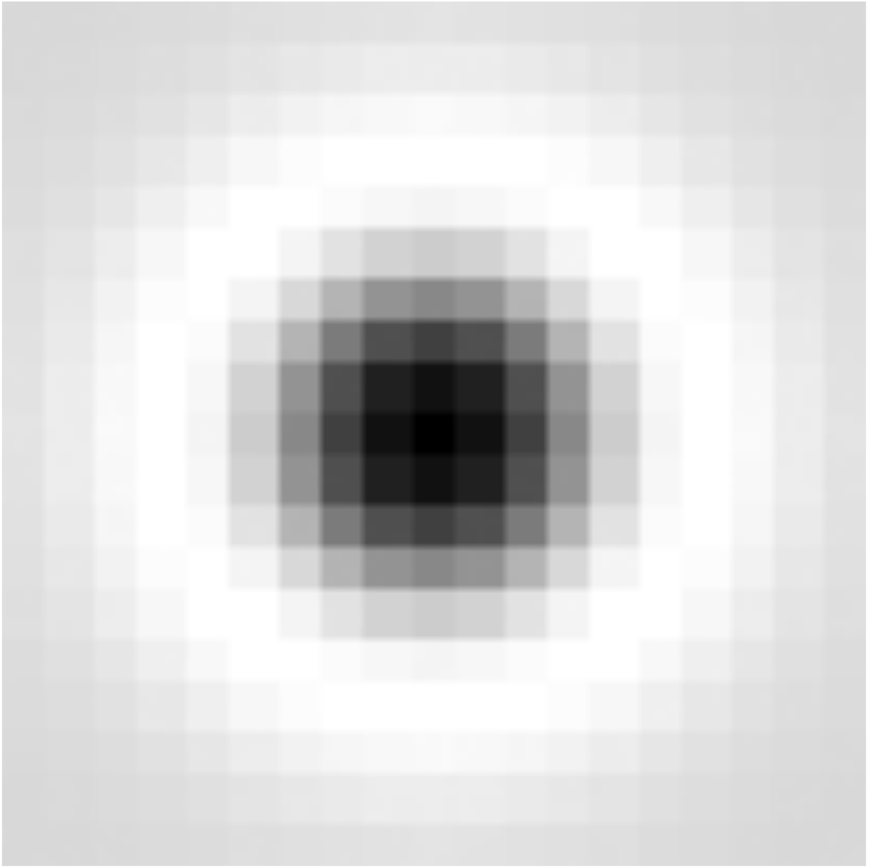


Source: N. Snavely

Blob filter

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

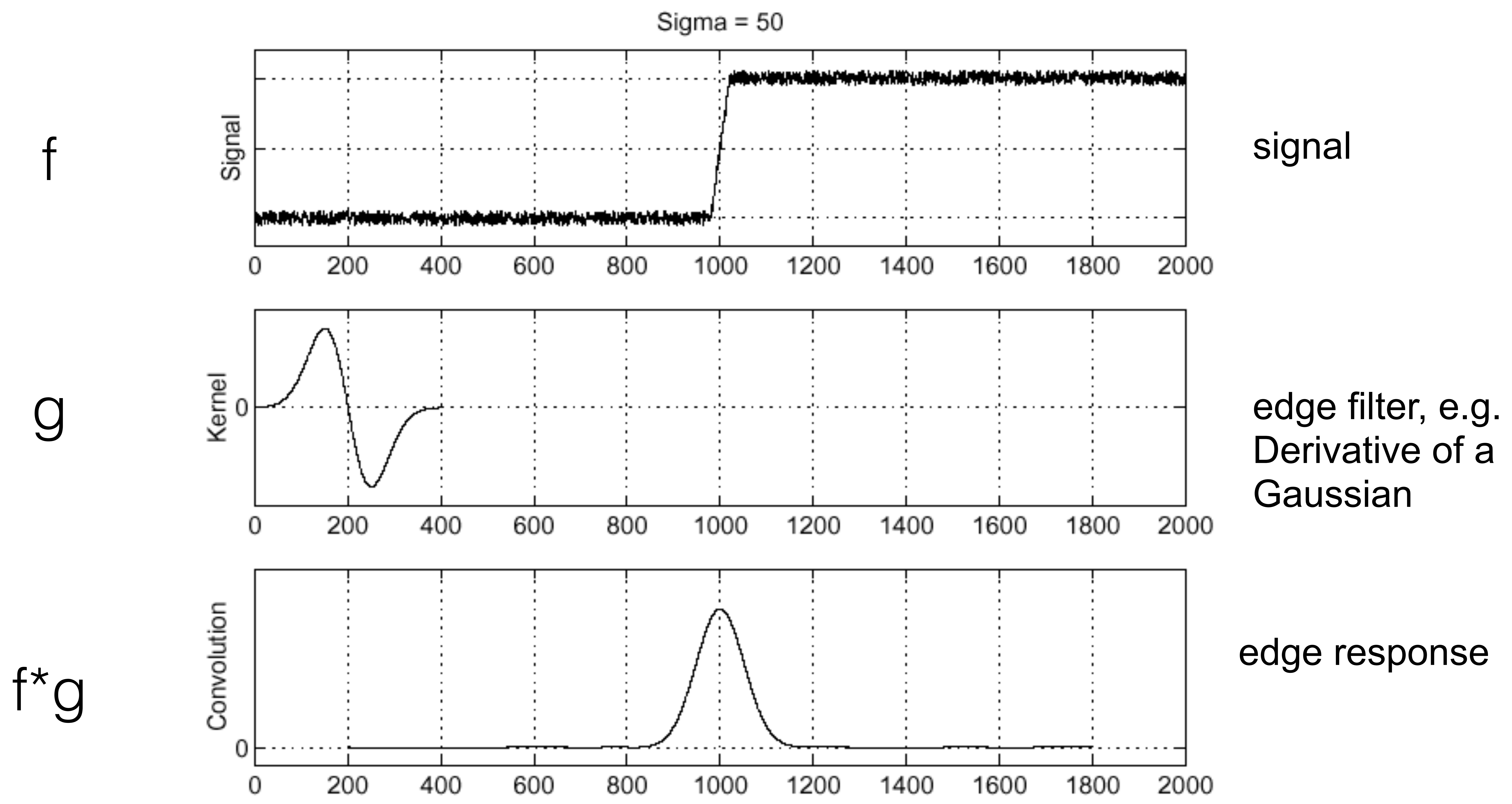




$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

Source: L. Lazebnik

Recall: edge detection

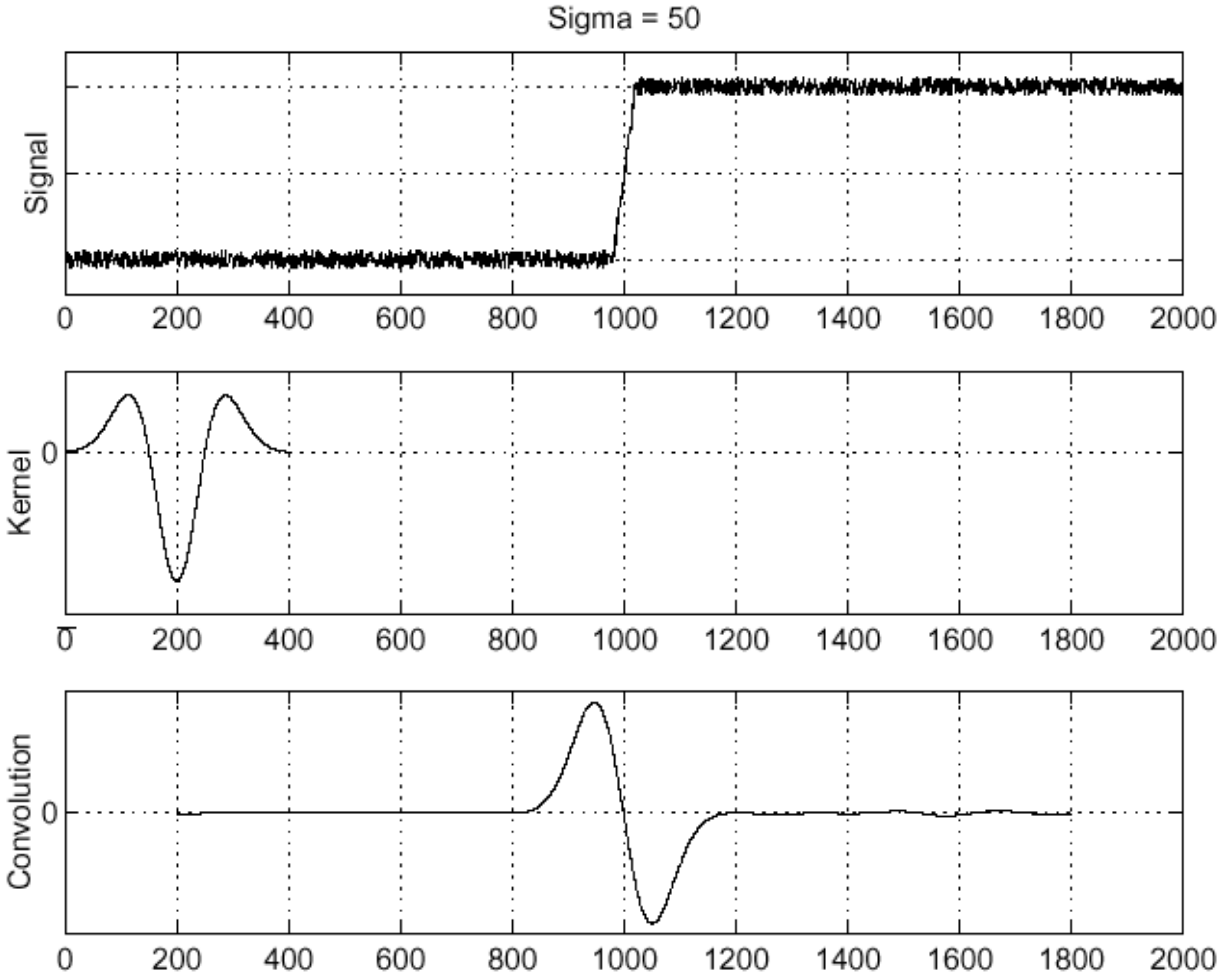


Edge detection using a Laplacian

f

$\frac{d^2}{dx^2} g$

$f * \frac{d^2}{dx^2} g$



Edge

Second derivative
of Gaussian
(Laplacian)

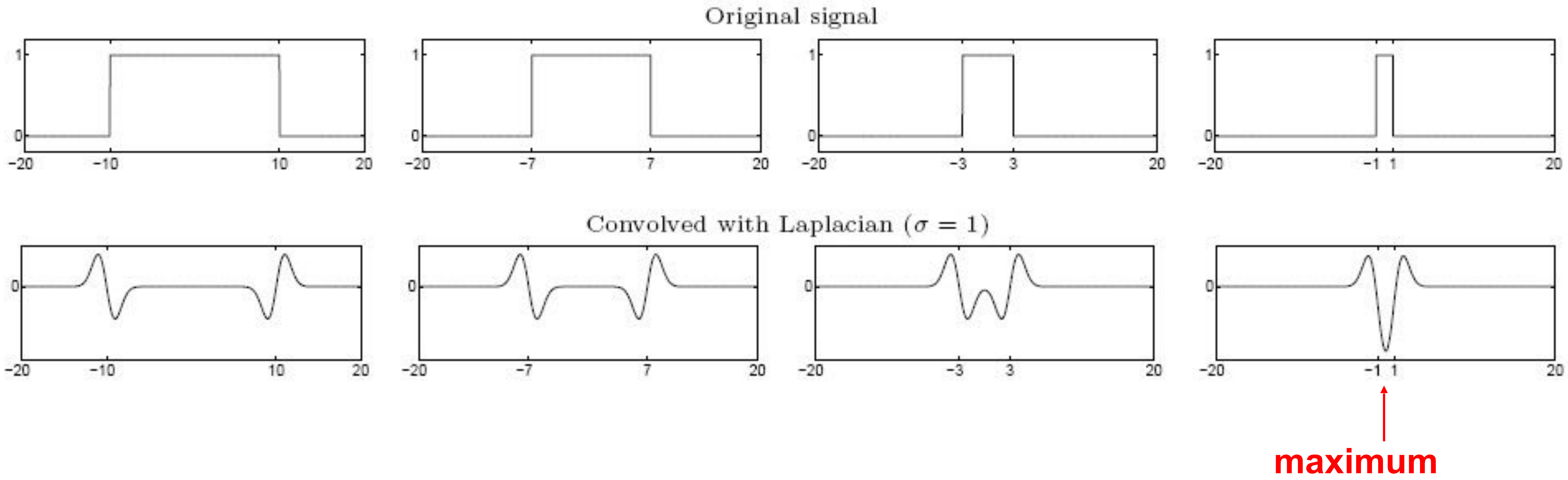
Edge = zero crossing
of second derivative

Source: S. Seitz

From edges to blobs

Edge — ripple

Blob — superposition of two ripples



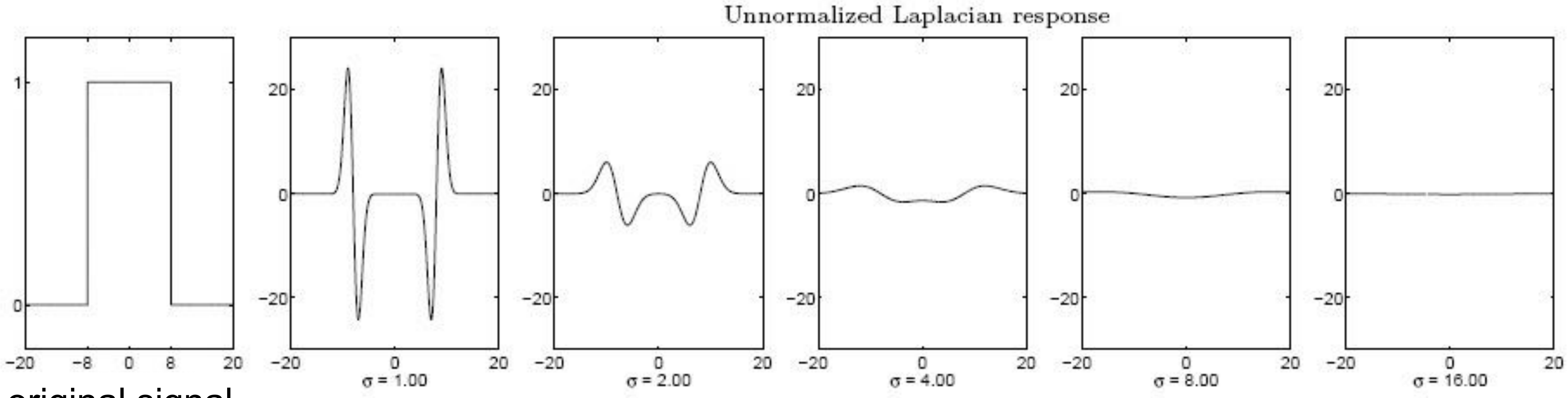
Spatial selection: the magnitude of the Laplacian response will achieve a maximum at the center of the blob, provided the scale of the Laplacian is “matched” to the scale of the blob

Source: L. Lazebnik

Scale selection

Find the characteristic scale of the blob by convolving it with Laplacians at several scales and looking for the maximum response

However, the Laplacian response decays as scale increases:



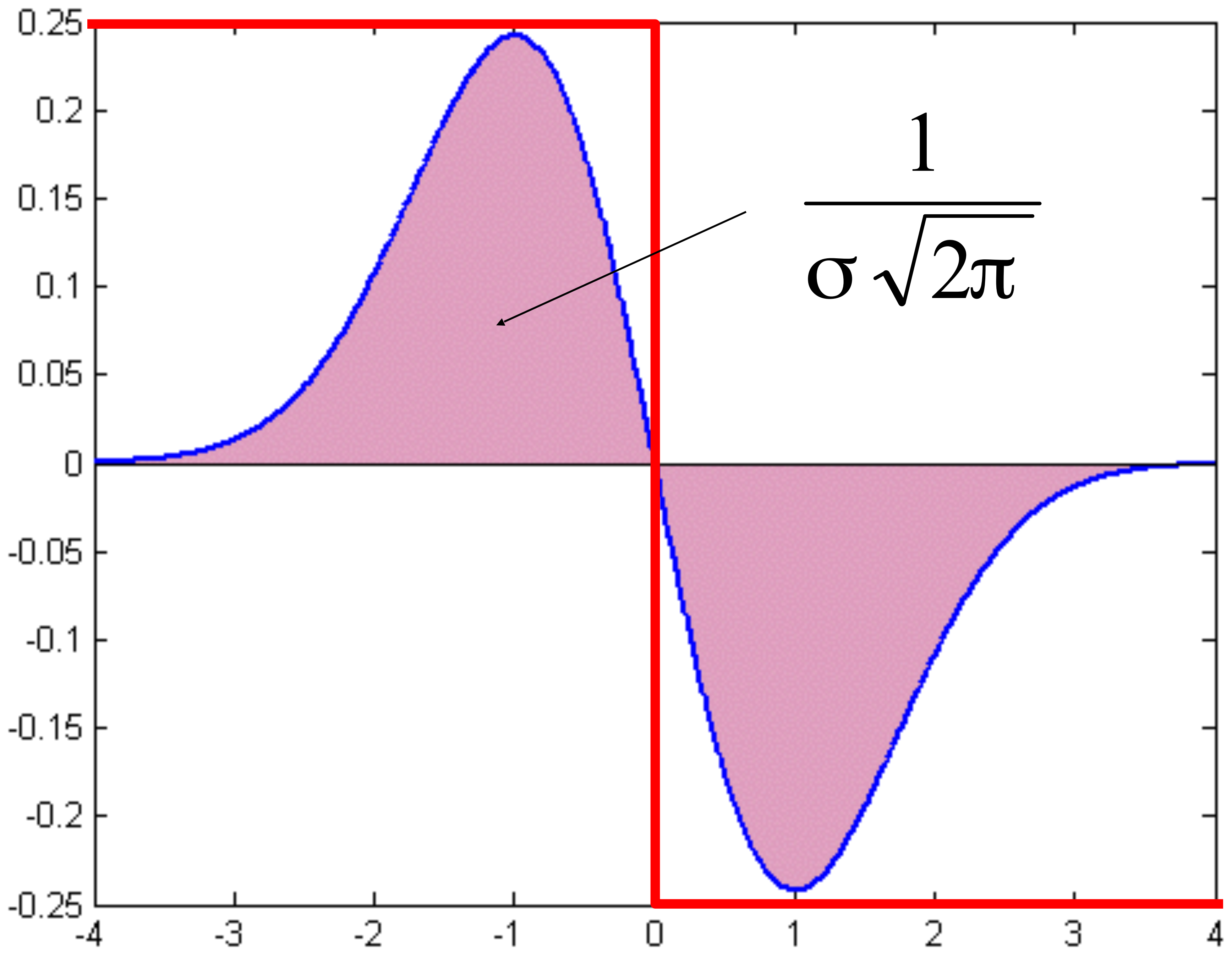
original signal
(radius=8)

increasing σ

Source: L. Lazebnik

Scale normalization

The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases



Source: L. Lazebnik

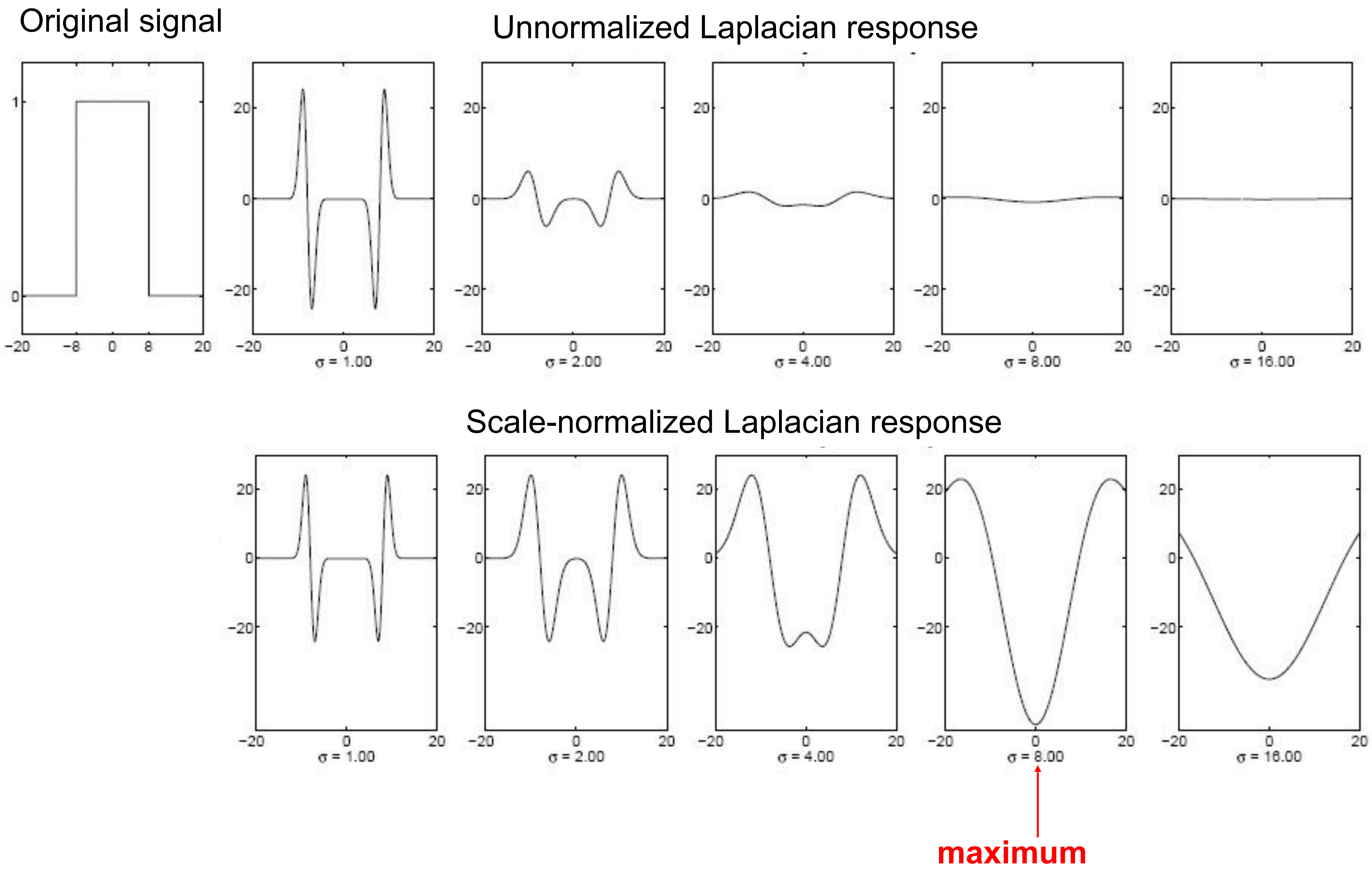
Scale normalization

The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases

To keep response the same (scale-invariant), must multiply Gaussian derivative by σ

Laplacian is the second Gaussian derivative, so it must be multiplied by σ^2

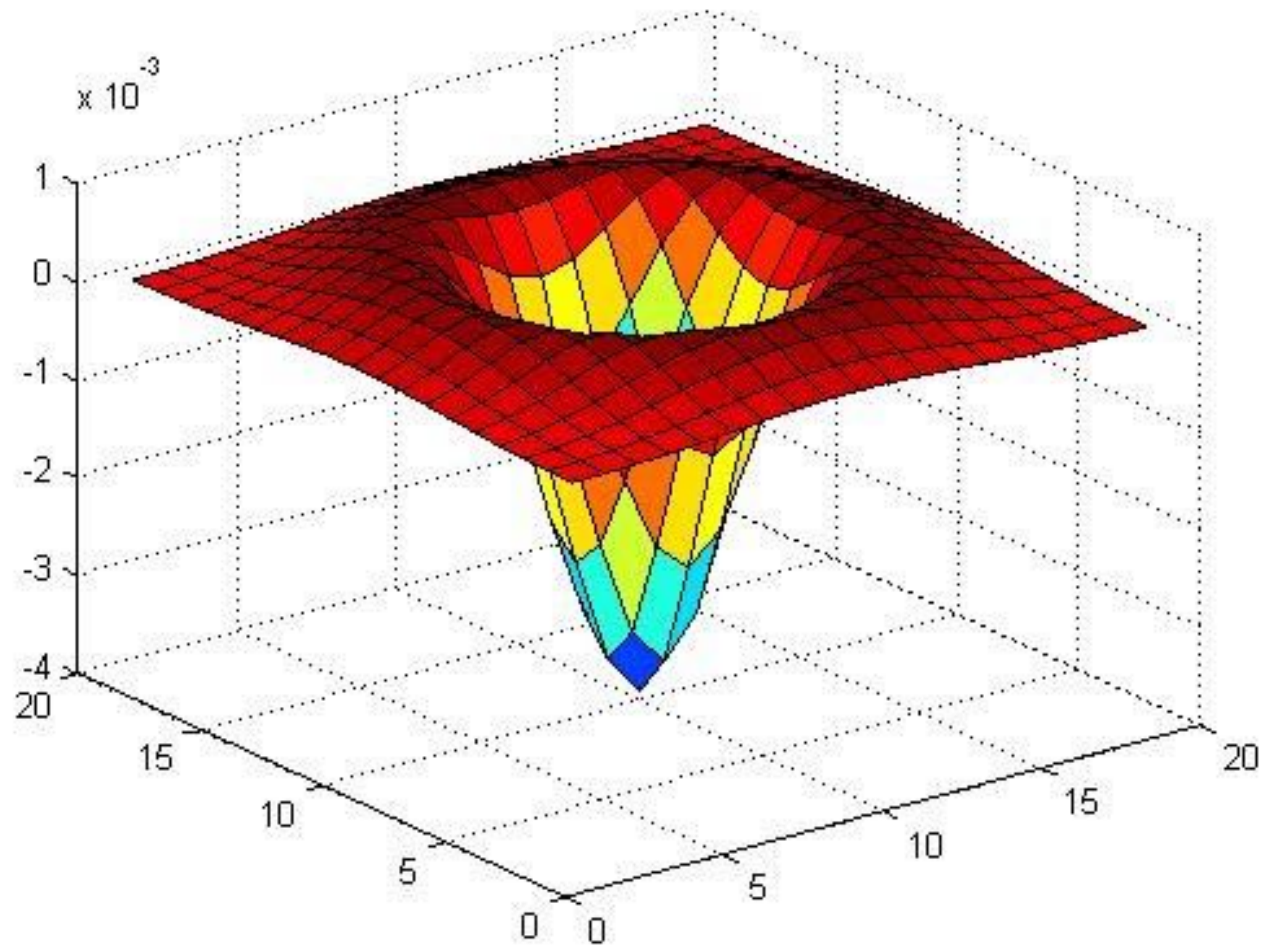
Effect of scale normalization

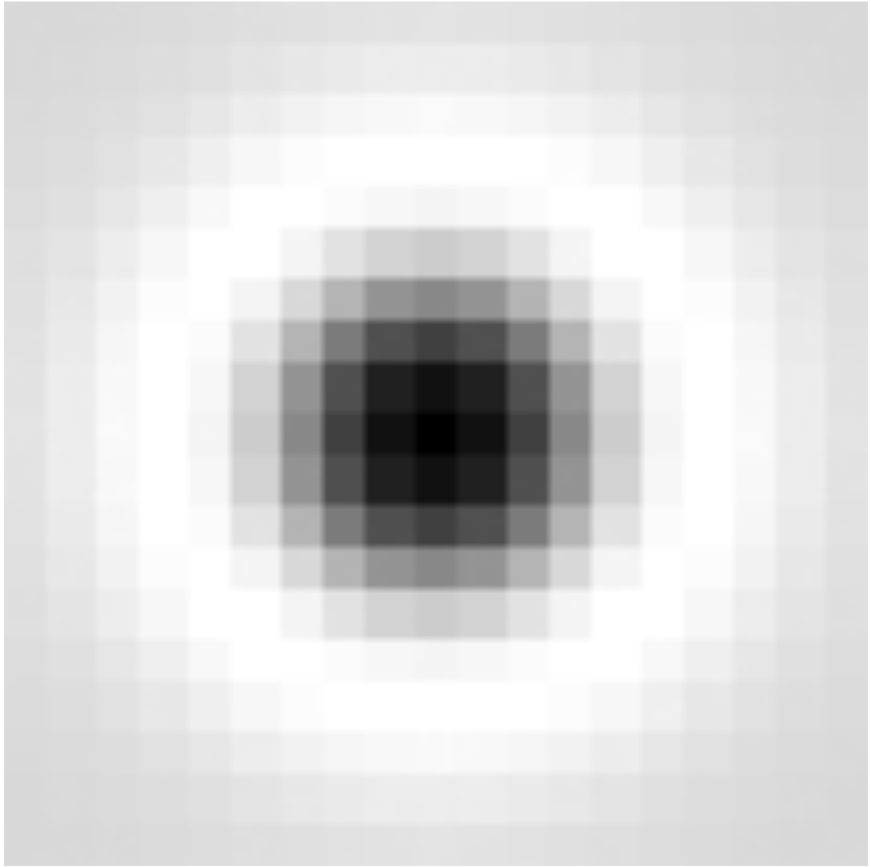


Source: L. Lazebnik

Blob detection in 2D

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D



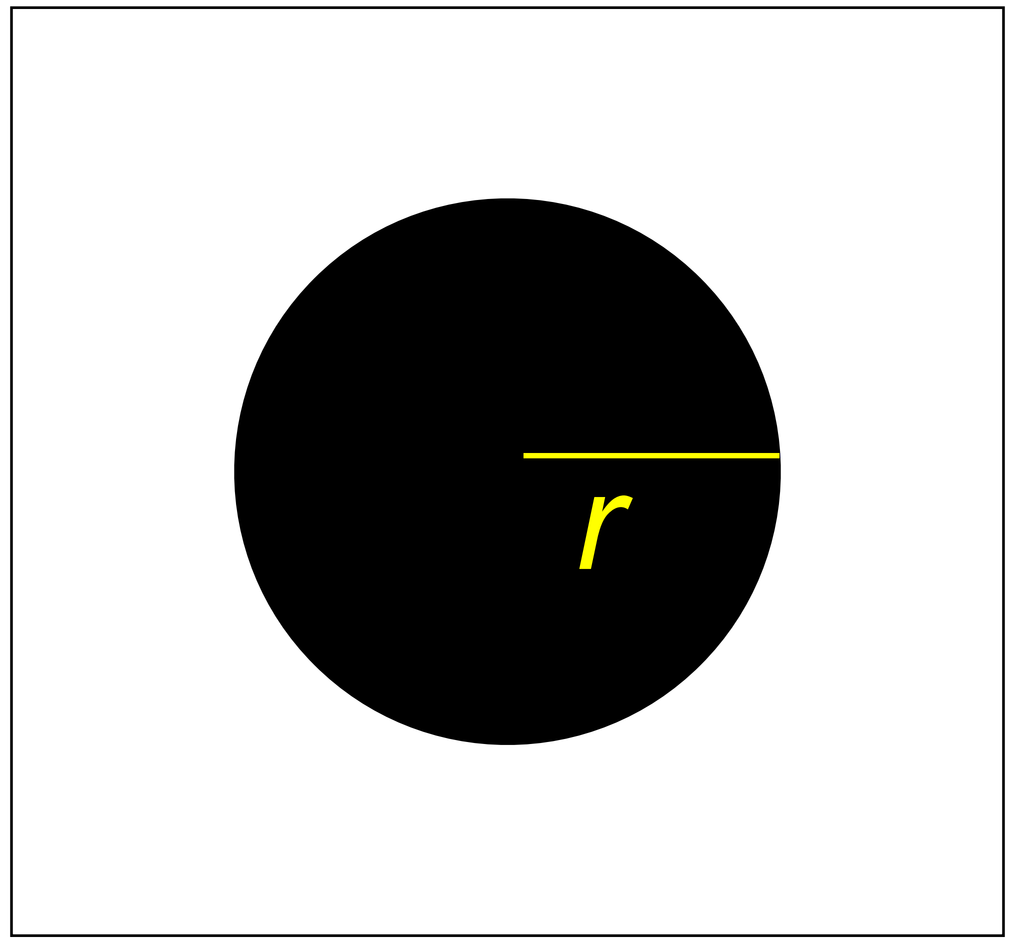


Scale-normalized: $$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

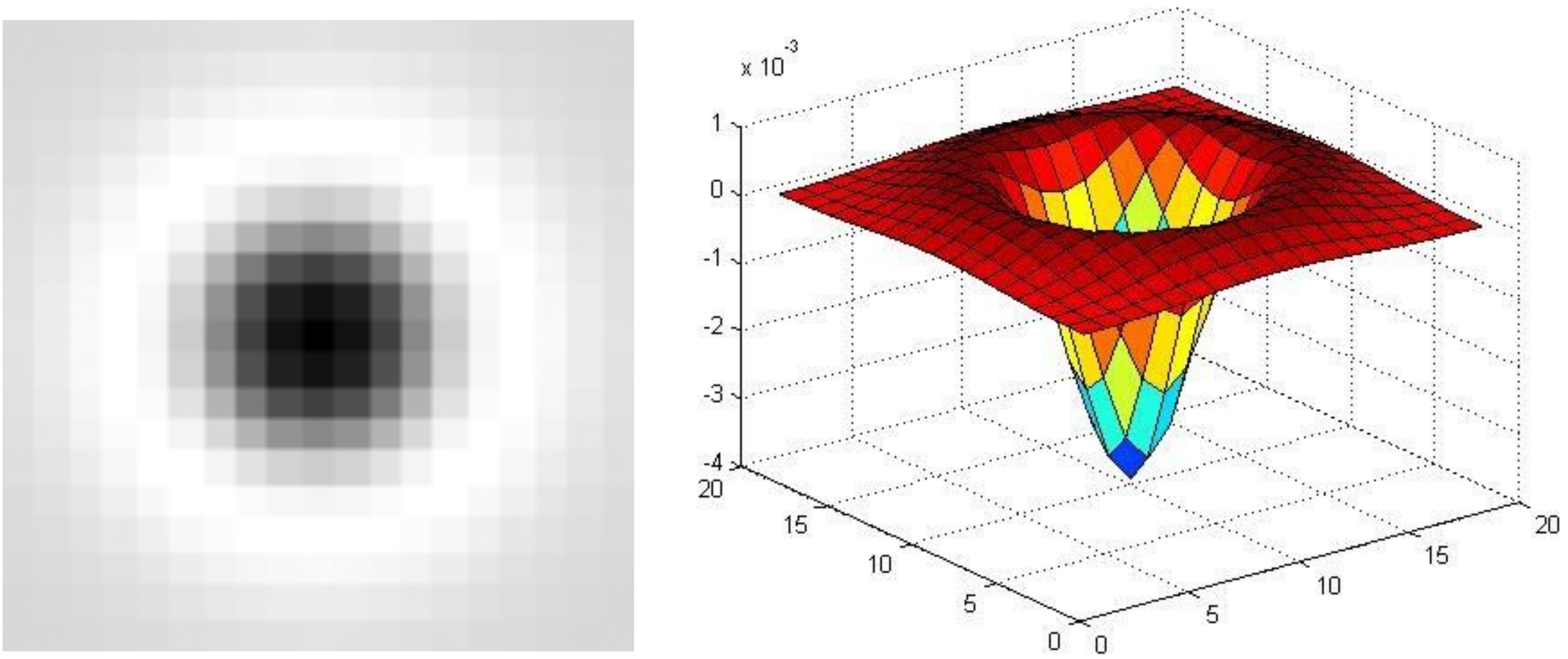
 Source: L. Lazebnik

Scale selection

At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?



image



Laplacian

Source: L. Lazebnik

Scale selection

At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?

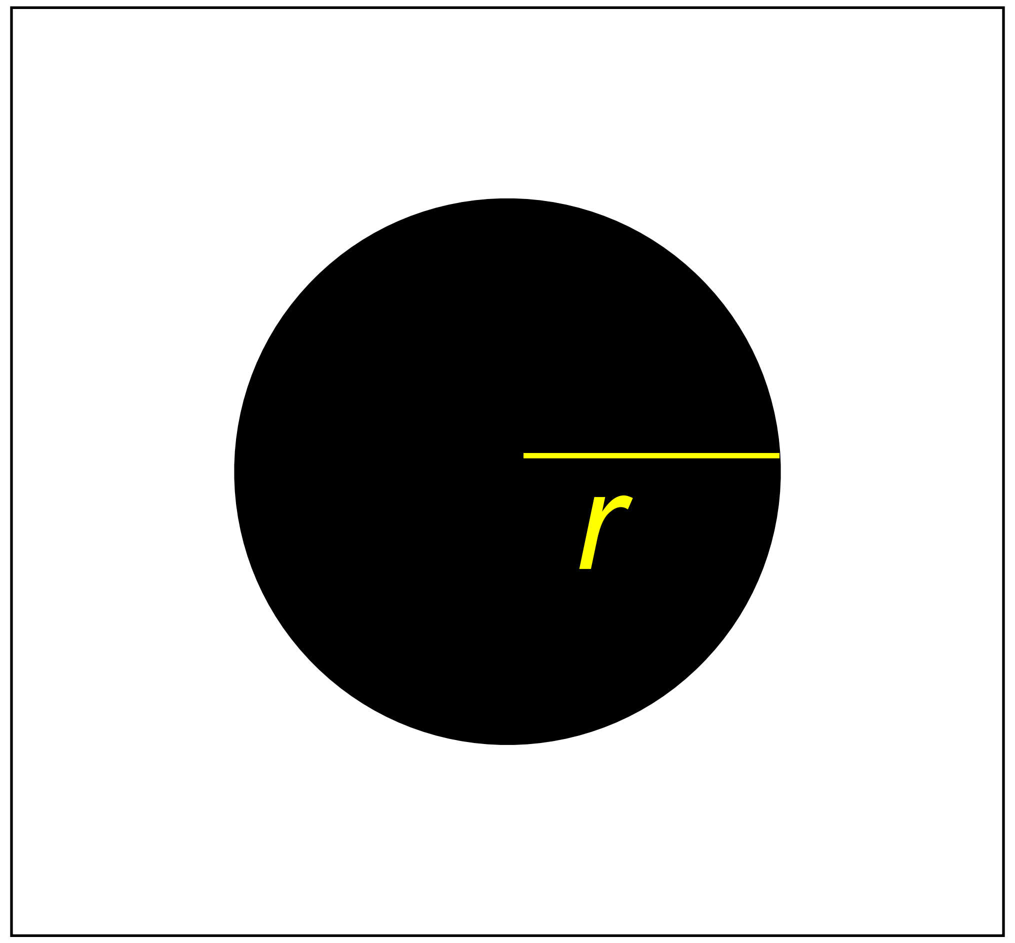
To get maximum response, the zeros of the Laplacian have to be aligned with the circle

The Laplacian is given by (up to scale):

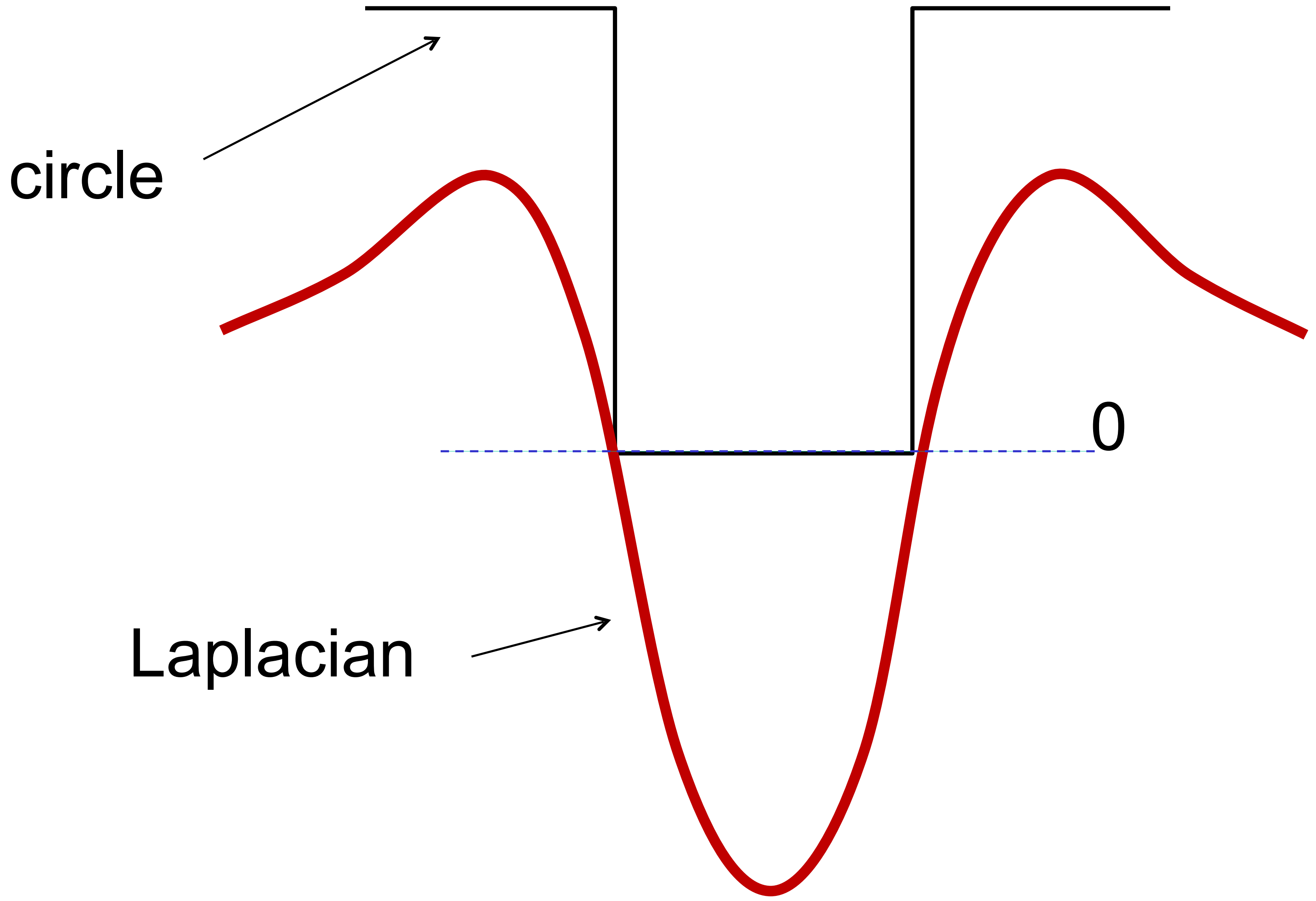
$$(x^2 + y^2 - 2\sigma^2)\, e^{-(x^2+y^2)/2\sigma^2}$$

Therefore, the maximum response occurs at

$$\sigma = r/\sqrt{2}.$$



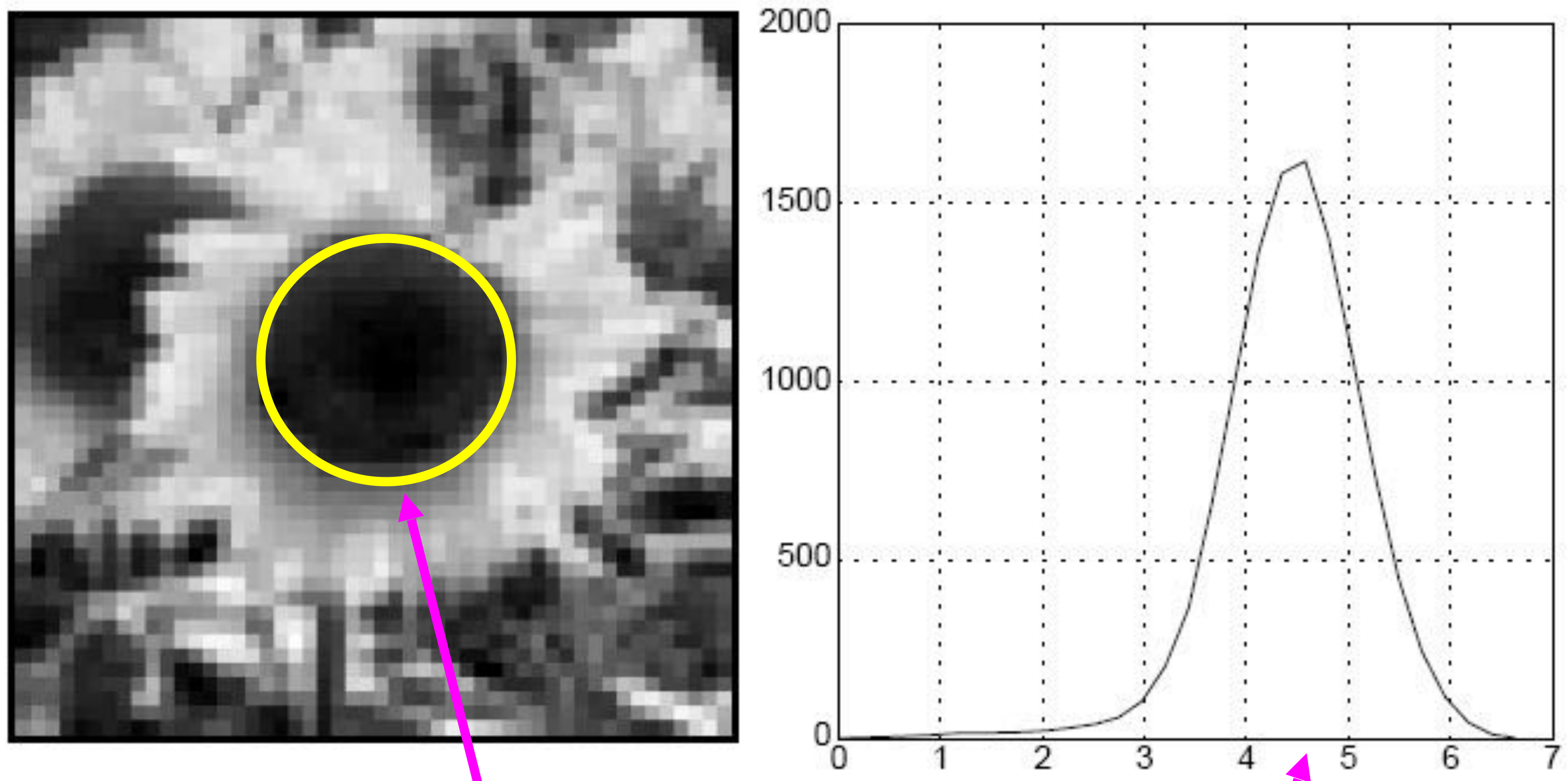
image



Source: L. Lazebnik

Characteristic scale

We define the characteristic scale of a blob as the scale that produces peak of Laplacian response in the blob center



characteristic scale

T. Lindeberg (1998). **["Feature detection with automatic scale selection."]** *International Journal of Computer Vision* **30** (2): pp 77--116.

Source: L. Lazebnik

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales

Source: L. Lazebnik

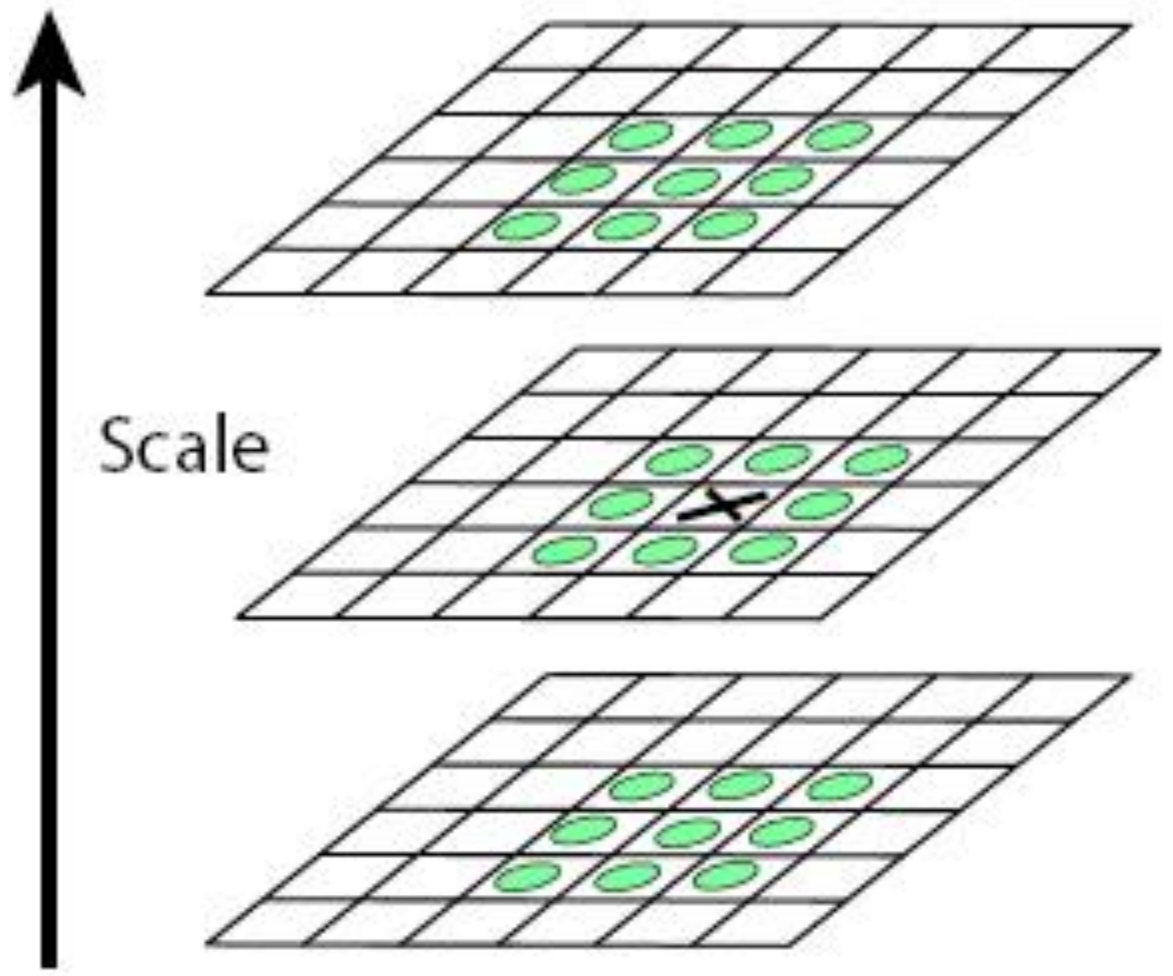
Scale-space blob detector: Example

Scale-space blob detector: Example

sigma = 11.9912

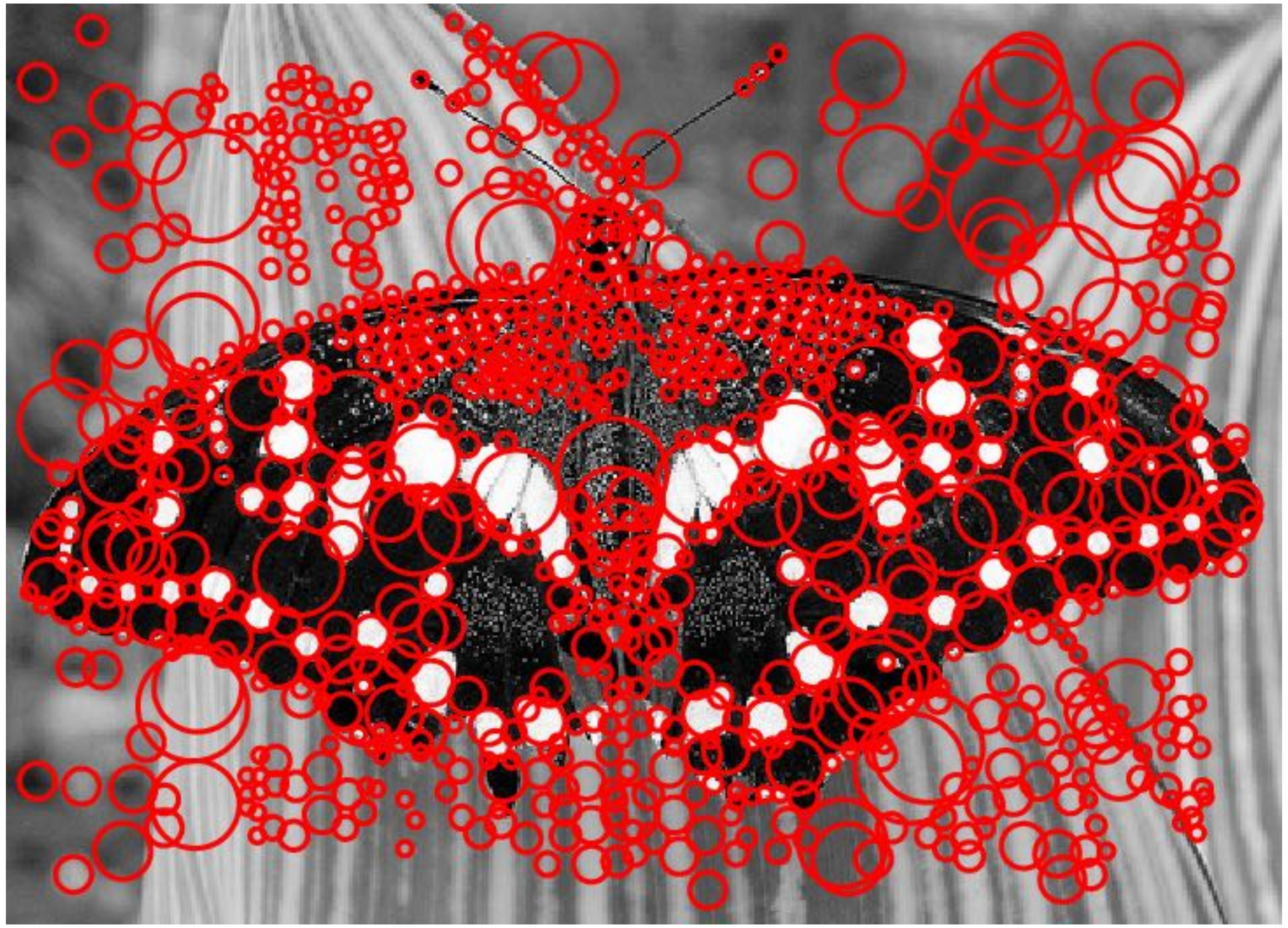
Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find maxima of squared Laplacian response in scale-space



Source: L. Lazebnik

Scale-space blob detector: Example



Source: L. Lazebnik

Efficient implementation

Is the Laplacian separable?

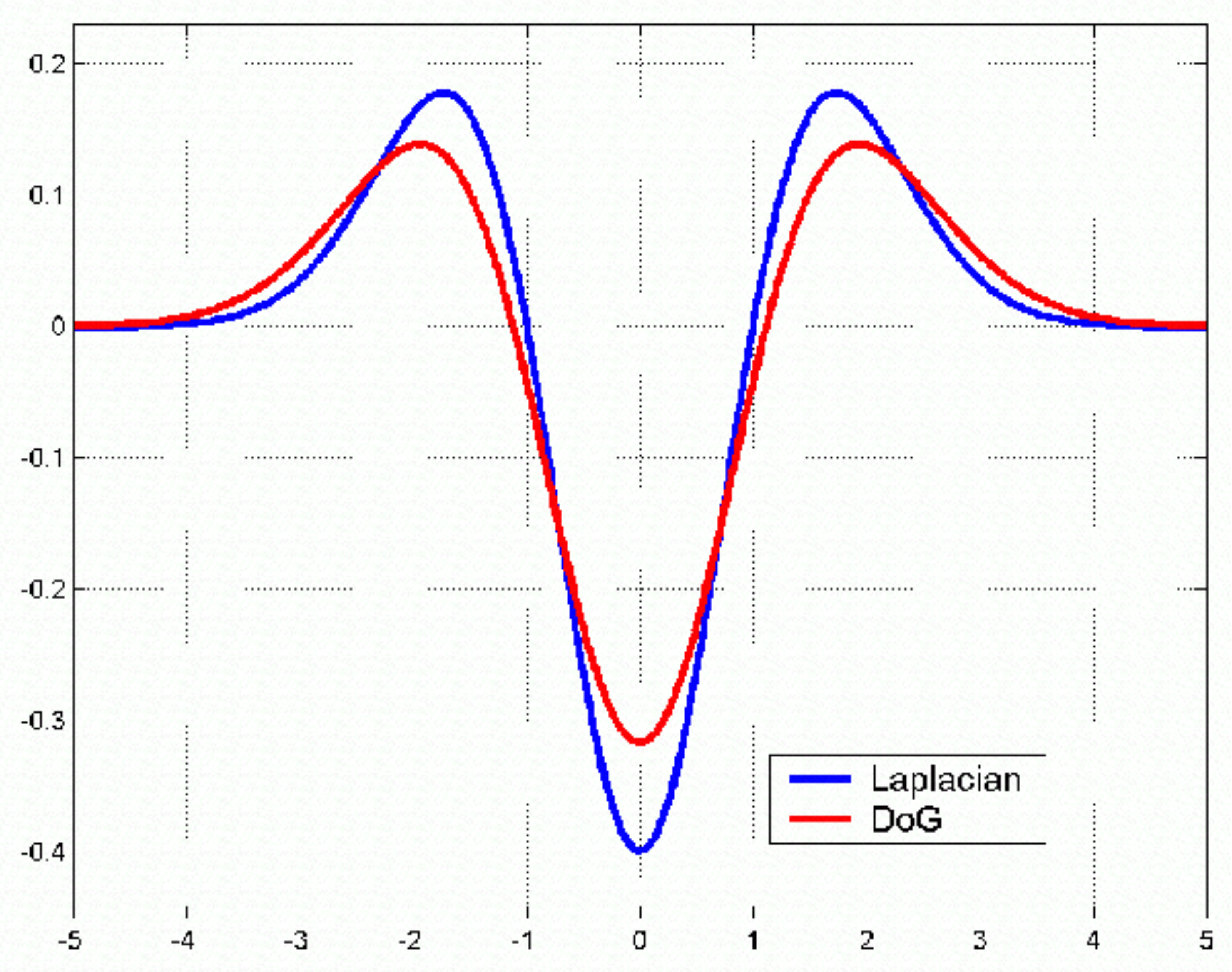
$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian)

Approximating the Laplacian with a difference of Gaussians:

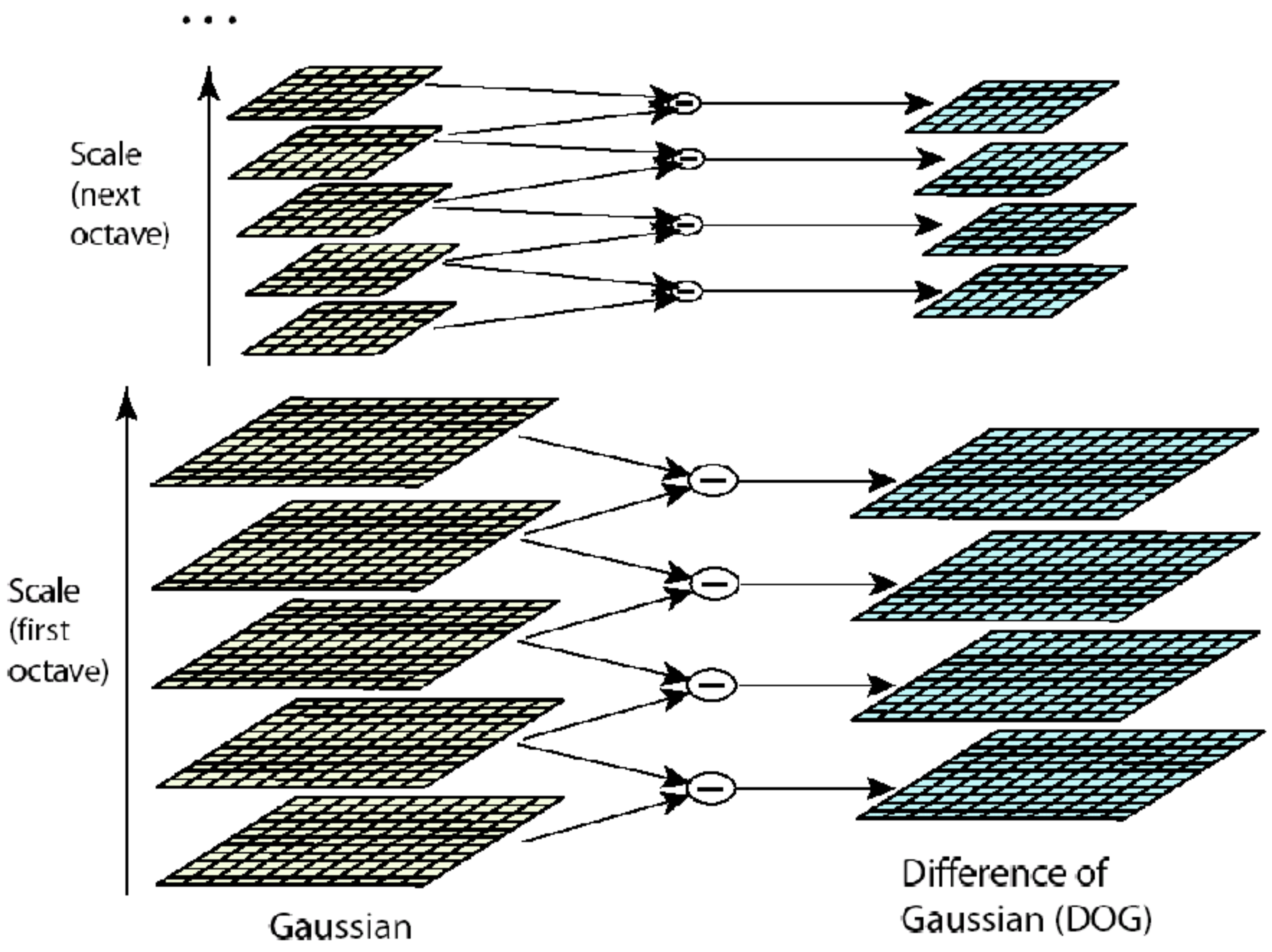
$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



Source: L. Lazebnik

Efficient implementation

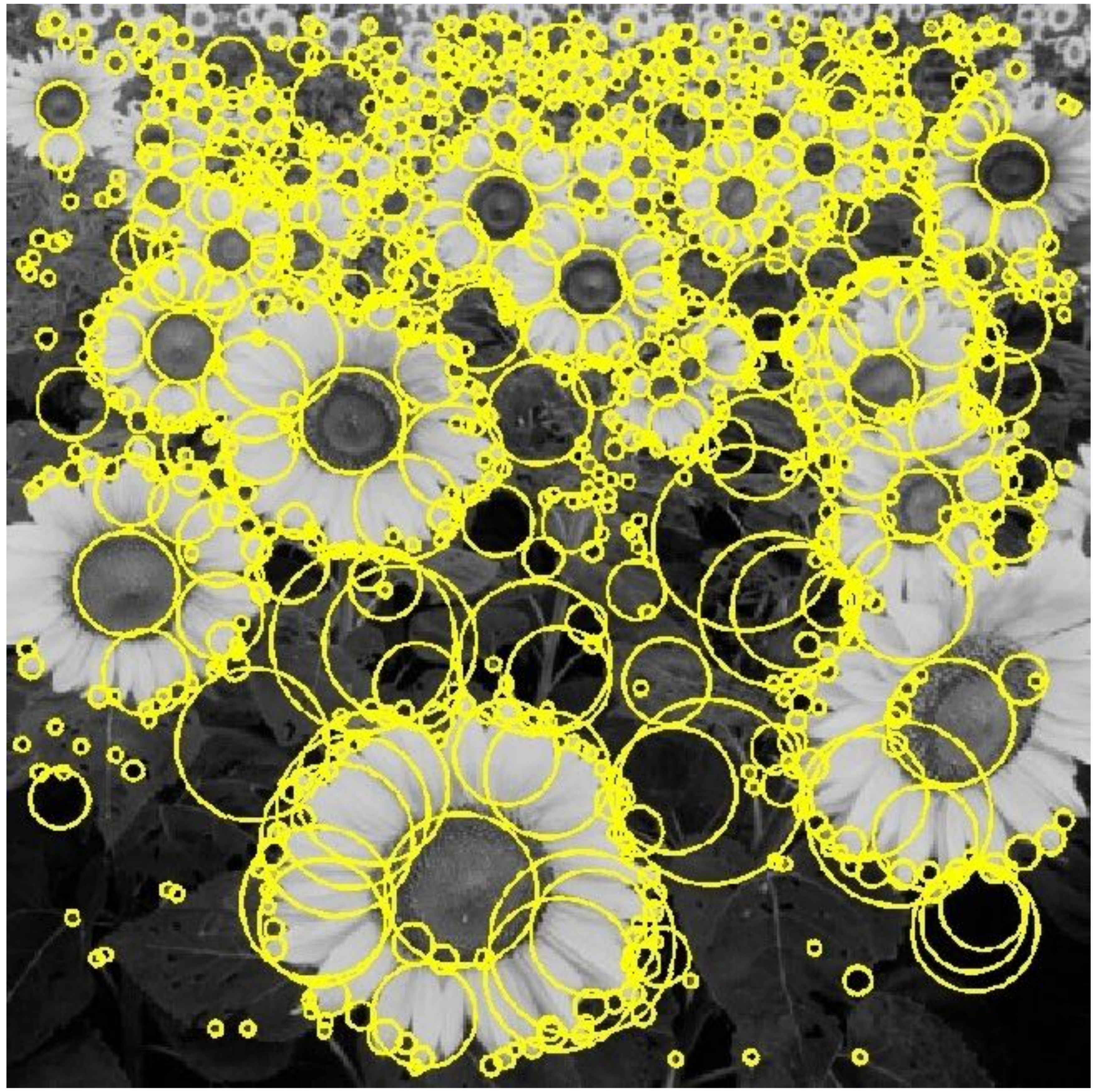


David G. Lowe. [**"Distinctive image features from scale-invariant keypoints."**]
IJCV 60 (2), pp. 91-110, 2004.

Scale covariant features

Scale “covariance”

“blob” detection



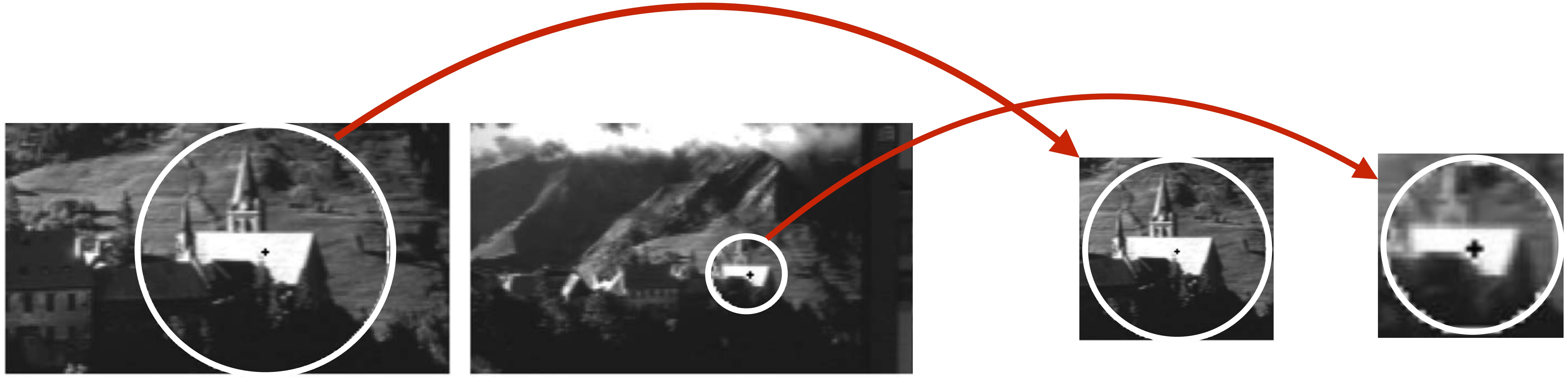
Source: L. Lazebnik

From feature detection to description

Scaled and translated versions of the same neighborhood will give rise to blobs that are related by the same transformation

What to do if we want to compare the appearance of these image regions?

- **Normalization**: transform these regions into same-size circles
- **Problem:** rotational ambiguity

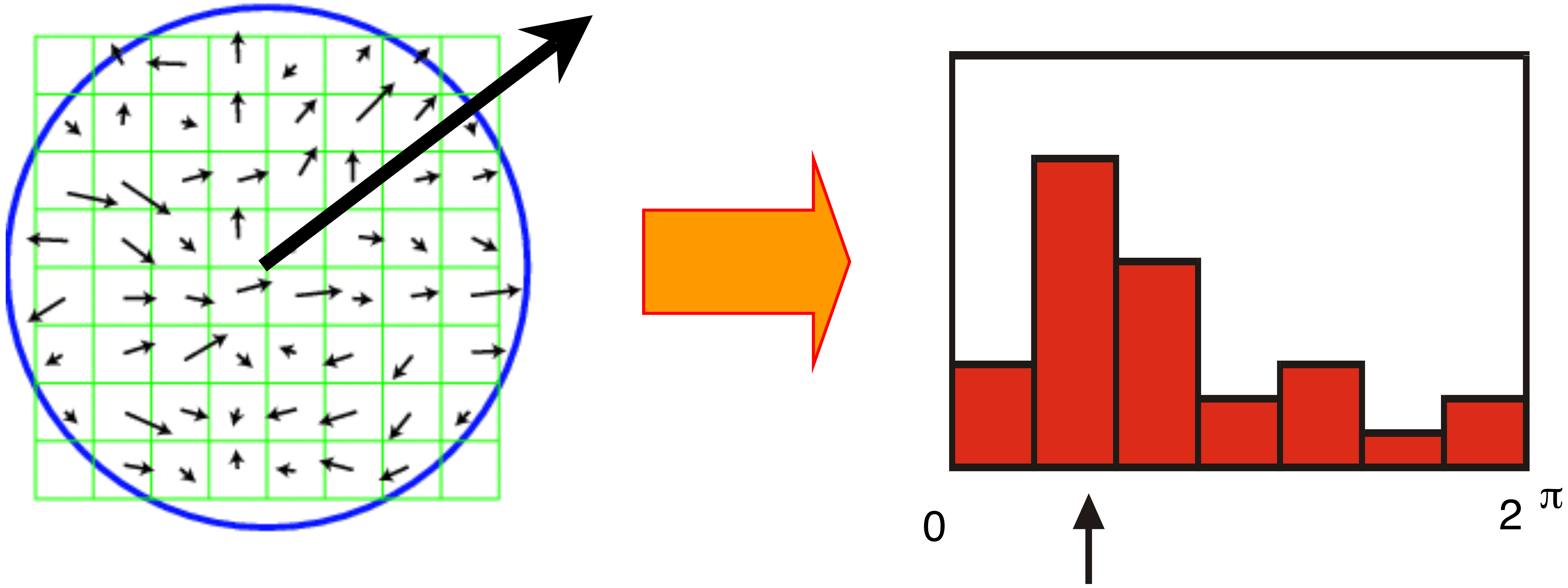


Source: L. Lazebnik

Eliminating rotation ambiguity

To assign a unique orientation to circular image windows:

- Create histogram of local gradient directions in the patch
- Assign canonical orientation at peak of smoothed histogram



Source: L. Lazebnik

SIFT features

Detected features with characteristic scales and orientations:

David G. Lowe. **["Distinctive image features from scale-invariant keypoints."](#)** *IJCV* 60 (2), pp. 91-110, 2004.

Source: L. Lazebnik

Run SIFT demo ….